POWER SPECTRUM OF HORIZONTAL WIND SPEED IN THE FREQUENCY RANGE FROM 0.0007 TO 900 CYCLES PER HOUR

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(Manuscript received 11 October 1956)

ABSTRACT

A power-spectrum analysis of horizontal wind speed is made over a wide range of frequencies by piecing together various portions of the spectrum. There appear to be two major eddy-energy peaks in the spectrum; one peak occurs at a period of about 4 days, and a second peak occurs at a period of about 1 minute. Between the two peaks, a broad spectral gap is centered at a frequency ranging from 1 to 10 cycles per hour. The spectral gap seems to exist under varying terrain and synoptic conditions.

1. Introduction

Power-spectrum analysis is a measure of the contribution of oscillations with continuously varying frequencies to the variance of a variable. Where wind speed is the variable, the variance is proportional to the kinetic energy of the speed fluctuations.

Panofsky and Van der Hoven [5] have shown that, for vertical motion near the ground, the major contribution to the total variance is within a frequency range from 10 to 1000 cy/hr. In the case of horizontal motion, the variance within that range is only a small portion of the total variance. By making a spectral analysis of 5-min average horizontal wind-speed components over a time interval of one week, these writers were able to show that a large amount of eddy energy existed in the frequency range between 10^{-2} and 10^{-1} cy/hr. Their principal conclusion was that there is a gap in the spectrum of horizontal wind velocity at a frequency of about 1 cy/hr. The resolution of the low-frequency end of this spectral study was limited, because the length of the time series that was used was only one week.

The aim of this article is twofold. First, a spectrum of horizontal wind speed is desired covering a broad enough frequency range so as to include all important contributions to the total variance. Second, it is desired to test the generality of the major peaks and gaps appearing in the spectrum under differing terrain and synoptic conditions.

[1] Research carried out under an agreement between the U. S. Weather Bureau and the U. S. Atomic Energy Commission.

2. Method of analysis

To obtain spectral estimates over a wide range of frequencies, two methods can be used. Either a lengthy time series of frequent readings covering several months is analyzed, thus giving the entire spectrum in one piece, or portions of the spectrum are analyzed separately and then pieced together. Because of the amount of data involved, the second method is the more practical one.

The procedure and problems of power-spectrum analysis over large frequency ranges are explained by Griffith *et al* [2]. The piecing-together of the various portions of the spectrum first involves a correction for the effect of the averaging of the individual readings. The use of the averaged rather than the instantaneous readings is necessary to avoid spurious results at high frequencies [9; 10], especially when the higher frequencies contain considerable energy

TABLE 1. Portions of horizontal wind-speed spectrum of fig. 1.

Height	Date and time	Reading average	Frequency range	No. of lags	Degrees of freedom*
108 m	6/25/55–4/30/56	5-day	0.0007–0.0042 cy/hr	6	19
108 m	8/9/55–2/25/56	1-day	0.0035–0.021 cy/hr	6	65
108 m	8/9/55–9/18/55	5-hr	0.01–0.1 cy/hr	10	38
108 m	0000, 8/20/55–0200, 8/22/55	½-hr	0.1–1.0 cy/hr	10	19
108 m	0000, 8/20/55–0200, 8/22/55	10-min	0.5–3.0 cy/hr	10	58
125 m	0730, 8/13/55–1400, 8/14/55	75-sec	0.8–24 cy/hr	30	96
91 m	0730, 8/13/55–0830, 8/13/55	20-sec	15–90 cy/hr	6	58
91 m	0730, 8/13/55–0830, 8/13/55	2-sec	30–900 cy/hr	30	118

* Number of unrestricted and independent variables entering into statistic. Tukey [8] has shown that spectrum estimates are distributed according to chi square divided by degrees of freedom, degrees of freedom being defined by $[2N - (3/2)m]/m$, where N is total number of observations and m is number of lags.

compared to the lower frequencies. A sufficient overlap of the spectral portions is also required in the procedure so that spurious results in the individual spectra due to aliasing [10] at the high-frequency end and poor resolution at the low-frequency end can be discarded.

3. Horizontal wind-speed spectra between 0.0007 and 900 cy/hr

The horizontal wind-speed data used in the spectral analysis of this section were obtained from *Aerovane* speed records at the upper three levels of the 125-m meteorological tower of the Brookhaven National Laboratory [7]. The data analyzed ranged from 5-day average speeds covering almost a year to 2-sec average speeds covering an hour. Table 1 gives the observation height, the date and time, the averaging period of the individual readings, the frequency range, the number of lag correlations in the analysis, and the degrees of freedom [8] of each of the spectral portions which were fitted together. The data were not available to make all the spectral analyses at one height level. Fig. 1 is a plot of the various spectra, each spectrum being represented by a different symbol. The abscissa is a logarithmic scale, with frequency indicated in cycles per hour and the corresponding period in hours. However, since one wishes to retain the property that the variance contributed within a frequency range is given by the area under the spectral curve, the original spectral estimates must be multiplied by the frequency [2; 4]. These quantities are plotted along the ordinate. The statistical significance of the major peaks and gaps of the spectrum is shown by a plot of the 5- and 95-percent fiducial limits [8].

The first major peak at the low-frequency end of the spectrum occurs at a period of about 100 hr (4 days). It is a very interesting fact that this corresponds to the period of the peak of the power spectrum of temperature as shown by Griffith *et al* [2]. Estoque [1] also found that the most important transfers of momentum and heat occur in the neighborhood of a period of 4 days. It seems reasonable, then, to conclude that the peak at a period of 4 days is the result of fluctuations in wind speed due to the passage of large, synoptic-scale pressure systems. This portion of the spectrum was obtained by an analysis of 24-hr average winds over an interval of 200 days (table 1). When divided into two equal portions of 100 days, each set of data still showed a distinct spectral peak of 5.1 and 4.2 m^2/sec^2, respectively, at a period of 4 days. Seasonal effects probably have an important influence on large-scale wind fluctuations, and therefore the 4-day energy peak would not be nearly so pronounced if one of the 100-day portions covered only the summer season.

The slight peak at a period of about 12 hr might be due to sampling fluctuations. However, according to the findings of Hellmann [3] on the diurnal variation of wind speed, a double maximum occurs at a height of 70 m, while a single 24-hr wind speed maximum is found at the ground and at a height of several hundred meters. Consequently, the fact that a height of about 100 m was chosen for the current study is probably the reason for the surprising lack of a 1-day spectral peak. The portion of the spectrum in fig. 1 showing the 12-hr peak was obtained from an analysis of 5-hr average speeds over a span of 40 days. When the data were analyzed in two separate 20-day portions, the slight peak appeared only in one portion. Except for this difference, the remaining spectral amplitudes of the two portions were very similar.

The spectral gap at about 1 cy/hr is located in the

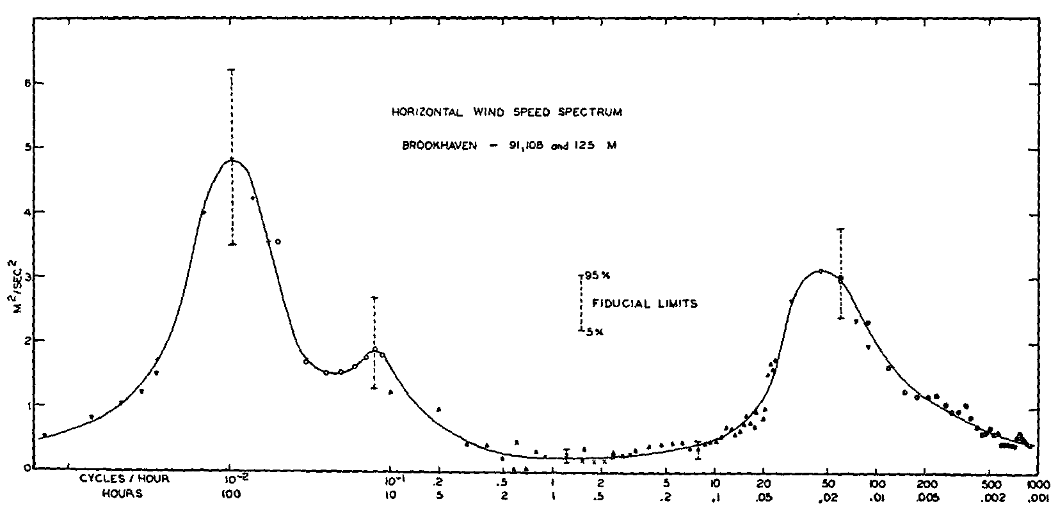

FIG. 1. Horizontal wind-speed spectrum at Brookhaven National Laboratory at about 100-m height. (See table 1 for date and time.)

same frequency range as that previously found by Panofsky and Van der Hoven [5]. The lack of a physical process which could support eddy energy in the atmosphere is thought to be the reason for the spectral gap in this range. The spectral analysis in this range is based on ½-hr and 10-min averages over a span of two days.

An anomalous wind situation was purposely chosen to represent the high-frequency end of the spectrum shown in fig. 1. The data were taken during hurricane Connie, with winds averaging 13 m/sec during a 30-hr interval and averaging 20 m/sec during the peak hour (0730 to 0830 EST 13 August 1955) of the hurricane. The effect of the high wind speeds was to increase greatly the amplitude of the high-frequency end of the spectrum. More normal situations, such as the 14 horizontal wind-speed spectra shown by Van der Hoven and Panofsky [11], seldom show a spectral amplitude above 1.0 m^2/sec^2 and none above 2.0 m^2/sec^2. Fig. 1 shows a peak amplitude of 3.1 m^2/sec^2 in this range.

If fig. 1 can be considered representative, the spectrum of horizontal wind speed seems to have two eddy-energy peaks, one at a period of 4 days and the other at a period of 1 min, with a spectral gap at a period of 1 hr. Very little energy is thought to be present outside the range shown in fig. 1. A measurement of the area under the spectral curve shows 60 per cent of the total variance at frequencies of less than 1 cy/hr, and 40 per cent at frequencies greater than 1 cy/hr. However, the relatively large percentage found at higher frequencies is peculiar to a very windy day.

4. Generality of spectral gap

Since the data used to describe the spectral gap cover such a short length of time, the question arises whether the gap might not be a sampling fluctuation. Furthermore, terrain features different from those at Brookhaven, or anomalous circulations such as those produced by thunderstorms, might reasonably provide the physical process which could serve as an eddy-energy source in this frequency range. This section of the study will test the generality of the spectral gap by an analysis of more data, both from Brookhaven and other sites with different terrain features.

Panofsky and Van der Hoven [6] showed that a horizontal wind-speed spectral gap exists over terrain different from Brookhaven by using wind-speed observations from an *Aerovane* located 30 ft above the roof of the 60-ft high Mineral Industries Building at University Park, Pennsylvania. The spectral analysis was made by using 1.25-, 5- and 20-min averages over a span of 48 hr. The terrain is typical of the ridge and valley section of the Appalachians. The spectral gap appeared to be centered at a frequency of about 2 cy/hr with an amplitude of 0.2 m^2/sec^2.

In this study, additional data were obtained from wind speed instruments at Oak Ridge, Tennessee, and Idaho Falls, Idaho. The Oak Ridge instrument is on top of a 100-m tower situated on the crest of a ridge. The Idaho Falls instrument is on top of a 76-m tower located on the Snake River Plain, with the Rocky Mountains rising 6000 ft above the plain 15 mi to the northwest. Figs. 2 through 5 show spectra of horizontal wind speed in the range from 0.4 to 24 cy/hr for two cases at each location.

Synoptic conditions during each of the cases were typical for the winter season. A continental polar air mass with no strong frontal passages predominated during the first Oak Ridge case (fig. 2). The second Oak Ridge case (fig. 3), which followed immediately after the first, began during the approach of another continental polar air mass, with a strong frontal

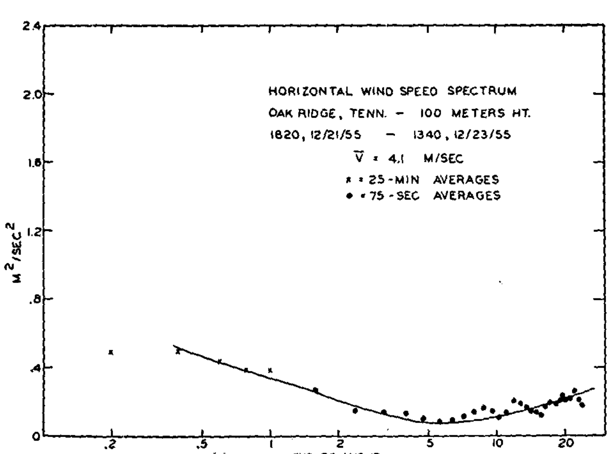

FIG. 2. Horizontal wind-speed spectrum at Oak Ridge at 100-m height during period 1820 EST 21 December 1955 to 1340 EST 23 December 1955.

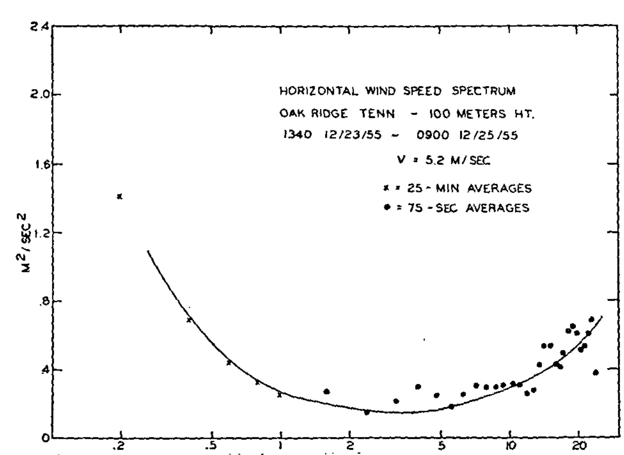

FIG. 3. Horizontal wind-speed spectrum at Oak Ridge at 100-m height during period 1340 EST 23 December 1955 to 0900 EST 25 December 1955.

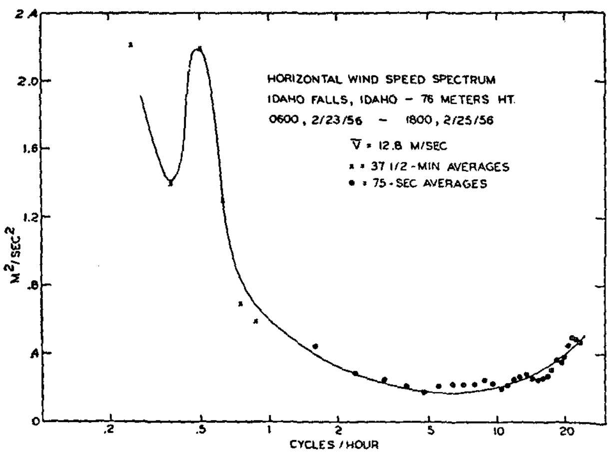

FIG. 4. Horizontal wind-speed spectrum at Idaho Falls at 76-m height during period 0600 MST 23 February 1956 to 1800 MST 25 February 1956.

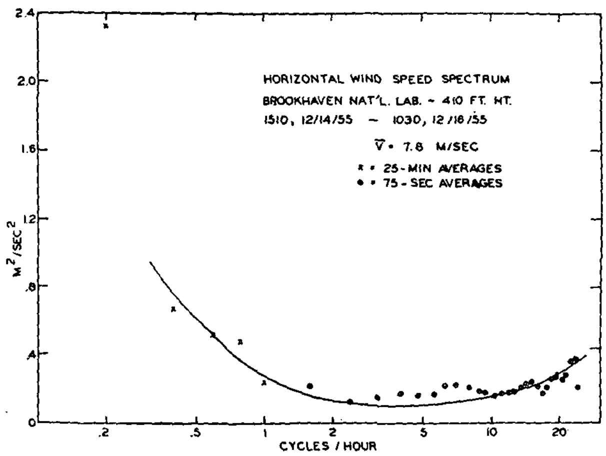

FIG. 6. Horizontal wind-speed spectrum at Brookhaven National Laboratory at 125-m height during period 1510 EST 14 December 1955 to 1030 EST 16 December 1955.

passage towards the end of the period. The first Idaho Falls case (fig. 4) was taken during maritime polar flow from the west, with a frontal passage in the beginning of the period. The maritime polar air mass continued to predominate during the succeeding second case (fig. 5), but with no major frontal passages and with weaker winds.

Each of the above cases shows a spectral gap in the range from 1 to 10 cy/hr with a spectral amplitude of between 0.1 and 0.2 m^2/sec^2. This is in general agreement with the spectrum of fig. 1. It is interesting to note that the strong winds accompanying the frontal passages during the second Oak Ridge case and the first Idaho Falls case (figs. 3 and 4) were associated with a markedly increased eddy energy at high frequencies (above 10 cy/hr). Both Idaho Falls cases showed a secondary spectral peak at 0.5 cy/hr. However, the significance level of this end of the spectrum is low. Therefore, a longer time series in this frequency range is needed to test the significance of the peak.

Another example was also obtained at Brookhaven during winter conditions (fig. 6). Continental polar air masses predominated, with a strong frontal passage during the period. In addition to the horizontal wind-speed spectrum, the spectrum of the east-west component of the horizontal velocity was also computed for this case and is shown in fig. 7. The agreement between these two spectral estimates is extremely close. The location and amplitude of the spectral gap agrees with the other cases.

A type of synoptic condition for which a spectral estimate has not yet been made at Brookhaven is the thunderstorm situation. A preliminary analysis of wind-velocity data during a thunderstorm period, by staff members of the Department of Meteorology at Pennsylvania State University, shows a spectral gap at a frequency of about 3 cy/hr with an amplitude of about 0.2 m^2/sec^2. There is also a small peak of 0.7 m^2/sec^2 at a frequency of 0.5 cy/hr.

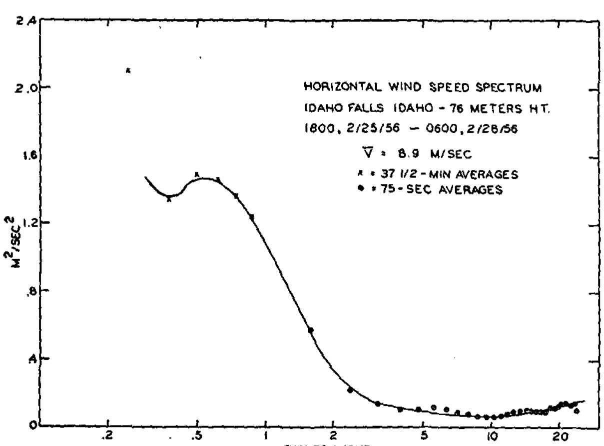

FIG. 5. Horizontal wind-speed spectrum at Idaho Falls at 76-m height during period 1800 MST 25 February 1956 to 0600 MST 28 February 1956.

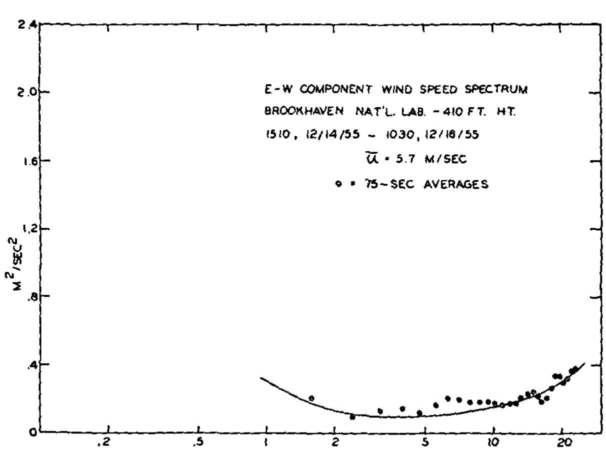

FIG. 7. East–west horizontal wind-speed spectrum at Brookhaven National Laboratory at 125-m height during period 1510 EST 14 December 1956 to 1030 EST 16 December 1956.

TABLE 2. Characteristics of horizontal wind-speed spectral gap.

Location	Dates	Ht. above ground (m)	$\bar{V}$ (m/sec)	Gap frequency (cy/hr)	Gap wave-number (cy/km)	Gap amplitude (m²/sec²)	Length of data (hr)	Degrees of freedom
Brookhaven	8/20–22/55	108	6.2	1	.04	.20	50	58
Brookhaven	12/14–16/55	125	7.8	3	.11	.15	43	114
Penn. State	6/4–6/55	30	1.8	2	.31	.15	48	114
Oak Ridge	12/21–23/55	100	4.1	6	.41	.10	43	137
Oak Ridge	12/23–25/55	100	5.2	3	.16	.15	43	137
Idaho Falls	2/23–25/56	76	12.8	5	.11	.20	60	190
Idaho Falls	2/25–28/56	76	8.9	9	.28	.10	60	190

5. Summary and conclusions

There appear to be two major eddy-energy contributions to the spectrum of horizontal wind speed; one peak occurs at a period of 4 days, and a second peak occurs at a period of 1 min. The former peak is due to wind-speed fluctuations caused by migratory pressure systems of synoptic weather-map scale. The latter peak is in the micrometeorological range and is a mechanical and convective type of turbulence [10]. No attempt was made to distinguish between the two types of horizontal wind-speed turbulence in the micrometeorological range. However, previous work [10] with vertical wind-speed spectra has given evidence that, in the case of marked convective activity, the two types of turbulence were distinguishable.

Between the two major eddy-energy peaks is a rather broad spectral gap centered at a frequency ranging from 1 to 10 cy/hr. The reason for the gap is thought to be the lack of a physical process that could support wind-speed fluctuations in this frequency range. Table 2 is a summarization of the characteristics of the spectral gap as found under varying terrain and synoptic conditions. These characteristics do not seem to indicate that the position of the spectral gap is dependent on mean wind speed or occurs at a constant wave number. There is an indication that the position of the gap is dependent on the severity of the terrain, the position being at higher frequencies with more rugged terrain. However, the physical basis for this relationship, if it exists, is not obvious. Despite the wide range of mean wind speeds and of terrain conditions, the amplitude of the gap seems to lie consistently between 0.1 and 0.2 m^2/sec^2. The facts that the gap amplitude is consistently low, and that the gap is quite broad and flat, do not make an exact determination of the center of the gap so vitally important.

Acknowledgments.—The writer wishes to thank the meteorological personnel at the Brookhaven National Laboratory, and the U. S. Weather Bureau offices at Oak Ridge and Idaho Falls, for making the wind-speed records available from which the spectral analyses were made. Thanks also is due Mr. James W. Cooley, of the Institute for Advanced Study, who arranged to have much of the computation work done on the Institute's electronic computer.

The writer wishes to express his gratitude to his colleagues in the U. S. Weather Bureau and at the Pennsylvania State University for their suggestions and critical reading of the manuscript, and to Mr. G. A. DeMarrais for his assistance in reducing the data and performing some of the computations.

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